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Experimental Friction Analysis Through Innovative Compound-Substrate Contact Modelling for Automotive Applications

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Abstract. The knowledge of the tire tread viscoelastic properties, deeply affecting the tire dynamic behavior, is a particularly crucial topic for tire manufacturers to achieve the optimal friction performance and to govern the degradation mechanism during the tire life cycle. To this purpose the use of a friction model, called GrETA. (Grip Estimation for Tire Applications), aiming to predict the friction coefficient arising at the tire-road interface under various possible operating conditions (temperature, pressure and relative velocity), is proposed. The GrETA. model requires both the roughness parameters, associated with micro- and macro-scale of the road profile, and the viscoelastic properties of tire tread compound, function of temperature and excitation frequency, in order to mathematically describe the contact phenomena between the rubber surface and the substrate countersurface. The compound characterization has been achieved employing a non-destructive methodology called VESevo, capable of estimating the viscoelastic Storage Modulus and Loss Factor, whereas the road profile characteristics have been optically acquired and synthetized. To validate the model outputs, a specific experimental campaign has been conducted employing an evolved version of the British Pendulum bench with different road profiles and compound specimens.

Keywords: Test bench, Friction, Tire viscoelasticity, Contact mechanics modelling.

INTRODUCTION

The role of tire mechanics is fundamental in the automotive field because it is strongly linked to the vehicle dynamics behavior. Indeed, the knowledge of tire viscoelastic properties and tire-road contact mechanism is crucial to optimize vehicle performance and safety. In the last decades, compound-substrate modelling has been deeply discussed in the scientific literature. Many theories have been developed to determine the frictional behavior of rubber sliding on a texture in a wide working range of conditions, experimented in terms of temperature, contact pressure, profile roughness, etc. The simplest case that can be analyzed is the contact between two spheres [1, 2]. Starting from this problem the contact sphere-plane can be deduced by imposing an infinite radius of curvature to one of the two spheres. More deeply oriented to tire/road interaction, Klüppel's and Persson's theories [3, 4, 5] are widely employed because they try to explain the contact modelling problem following approaches and hypothesis linked to the characteristics of viscoelastic materials and of hard and randomly rough surfaces. The identification of the macro-roughness scales and especially of the micro-scales is an enigma yet to be unequivocally solved for the optimization of the multiscale theories [6]. The method used by Klüppel and Heinrich [3, 7] refers to the fractal scaling behavior of many rough substrates and to the linear viscoelastic response of the rubber [8]. Particularly, it considers hysteresis and adhesion contributions during sliding on rough substrates, whereby micro and macro roughness are taken. Persson has developed a theory that allows a certain number of lengths to be considered to define the friction coefficient. One of the main topics that Persson has analyzed is the nature of the area of real contact and it is important to understand his contact theory [9]. The early theories of Persson have stated that the adhesive rubber-substrate interaction can be negligible due to the low distance cut-off in the sum over length scales, which is larger than 0.1 μm [10]. However, many studies have been carried out to analyze in-depth the adhesive contribution, which unfortunately remains even more radically empirical [3, 11]. The most recent empirical model

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proposed by Persson is quite similar to the Klüppel's and assumes friction as proportional to the shear stress, which is a Gaussian-like curve as a function of the logarithm sliding velocity [6, 12]. On the other side, the properties of the rubber compound are complex to determine, unless the tire tread has usually to be destroyed to obtain a specimen tested with Dynamic Mechanical Analysis (DMA), which usually requires expensive machines and a long time for a full time-temperature characterization of the material according to William-Landel-Ferry theory [13, 14]. Nonetheless, in most applications, the possibility to perform tires laboratory testing, especially destructive ones, is restricted, especially in motorsport environments due to the competition regulation.

In this paper, the authors' aim is to characterize the tire tread compound through innovative non-destructive viscoelastic analysis and device [15], and through a multi-contact simplified physical model, called GrETA (Grip Estimation for Tire Analyses), for the analysis of adhesive and hysteretic contributions to the overall friction between rubber and substrate macro and micro asperities. In conclusion, the results obtained in terms of friction coefficient from the physical model will be compared to the experimental one, obtained from a test bench called BP EVO [16], in alternative to this friction measurement system, another of those cited in literature can be applied [17].

GRETA (GRIP ESTIMATION FOR TIRE ANALYSES) MATHEMATICAL MODEL

The rubber friction is a phenomenon influenced by different variables, which are often hard and difficult to be measured; it depends on the road macro and micro roughness, on the contact pressure arising in the footprint, on the rubber viscoelastic properties, on sliding velocity, frequency, and temperature [18]. Therefore, the GrETA model requires three main clusters of input variables to feed up the algorithms for the grip estimation:

- Roughness parameters: spatial frequency of macro asperities (λ_{macro}), average roughness (R_a), Hurst coefficient (H) and magnification level (ξ) are determined by means of HDC or PSD analyses.
- Compound viscoelastic properties of tire tread: storage modulus (E_1), loss factor ($\tan(\delta)$) and WLF parameters (C_1, C_2, T_0) thanks to VESevo non-destructive analysis.
- Simulation operating conditions: nominal contact pressure (σ_0); compound temperature (T_{sim}) and sliding velocity (v_{ssim}).

Model description and assumption

To model the complex interactions between the tires and the asphalt at a microscopic level, it was necessary to focus initially on the behaviour of an elementary volume of rubber in sliding contact with a limited portion of the road. The GrETA model is based on the following assumptions:

- The road asperity is modelled as a periodic axisymmetric rigid indenter with amplitude equal to R_a and frequency λ_{macro} in sliding contact with the tire block:

$$z(x,y) = R_a + R_a \cos\left(\frac{2\pi}{\lambda_{macro}} x\right) \cos\left(\frac{2\pi}{\lambda_{macro}} y\right) \tag{1}$$

- A generic sinusoidal indenter, corresponding to a wavelength between the macro and micro range, is defined by equation (2), where H is the Hurst coefficient and ξ_i the i-th magnification level, on which depend the generic λ_i and scaled R_a according to the self-affine profiles' theory [9, 19, 20, 21, 22, 23].

$$z(x,y) = R_a + R_a \cos\left(\frac{2\pi}{\lambda_{macro}} x\right) \cos z(x,y) = R_a + R_a \cos\left(\frac{2\pi}{\lambda_{macro}} x\right) + R_a \cos\left(\frac{2\pi}{\lambda_{macro}} y\right) + \frac{R_a}{\xi_i^H} \cos\left(\frac{2\pi}{\lambda_i} x\right) + \frac{R_a}{\xi_i^H} \cos\left(\frac{2\pi}{\lambda_i} y\right)\left(\frac{2\pi}{\lambda_{macro}} y\right) \tag{2}$$

- The tire tread block in contact with the sinusoidal road asperity is modelled as a homogeneous and isotropic parallelepiped. The nodes at the boundary of the parallelepiped are fixed in space. This implies a slight approximation respect to the real conditions which will be explained in detailed in the next paragraph. The parallelepiped dimensions depend on the considered wavelength and the tread thickness t as follows:

$$V = \lambda_i \times \lambda_i \times t \tag{3}$$

- The contact between the road sinusoidal indenter and the tire tread block is studied by means of Hertz theory. Particularly, the indenter is approximated to a sphere, whose radius R depends on the roughness parameters (R_a and λ) as expressed in equation (4), being the assumption of small deformation valid for compounds of passenger or track tires:

$$R_i = \frac{\lambda_i^2}{4\pi^2 R_{a_i}} \tag{4}$$

R_{ai} = roughness of the i-th wavelength considered
λ_i = macroroughness of the i-th wavelength considered
The applied normal load, F_i, is equal to:

$$F_i = \sigma_0 \lambda_{macro}^2 N_{C_i} \tag{5}$$

Where σ_0 is the simulation nominal pressure and N_{C_i} is the number of contacts between the tread block and the roughness asperities corresponding to a spatial frequency λ_i.

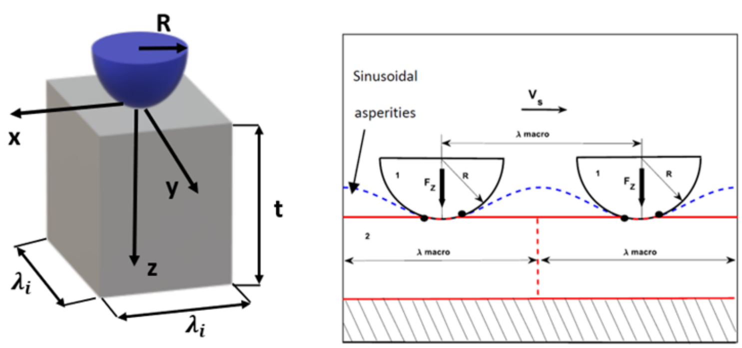
The number of contacts at i-th wavelength within the compound area λ_{i2} can be iteratively evaluated as follows:

$$N_{C_i} = \frac{\pi a_{i-1}^2}{\lambda_i^2} N_{C_{i-1}} \tag{6}$$

a = contact radius evaluated thanks to the Hertzian contact theory

The number of contacts assigned on the macro wavelength is approximated to 1:

$$N_{C_{i=1}} = N_{C_{macro}} = \frac{\pi a_{macro}^2}{\lambda_{macro}^2} \sim 1 \tag{7}$$



a) b)

FIGURE 1. a) Indenter-rubber volume representation – b) Sinusoidal road indentation

Further assumptions must be considered before evaluating the stress field distribution:

- TThe stress values are determined with respect to the assigned reference system in Fig. 1.a). The coordinates x, y and z are dimensioned with respect to the contact radius relative to the wavelength considered.
- To account for the simultaneous effect of several adjacent road asperities, five indenters are supposed to work.
- To decrease the error related to the balance between applied load and induced stress field, the thickness t is scaled on the number of contacts NCi occurring in the tread volume area $\lambda i2$ in the plane XY. However, it is necessary to highlight that the Sackfield-Hills relationship [1] for stress calculation is valid in case of contact between the sphere and infinite plane of the substrate: the algorithm cannot be applied on infinite volume domain, therefore a good compromise is achieved by choosing the domain limits showed on Fig. 1 a).

Adhesive Model

The adhesive grip contribution, linked to the local bonds between the tread block and the road micro asperities, is evaluated using the generalized formulation of the Klüppel's theory [3]:

$$\mu_{adh} = \frac{F_{adh}}{F_N} = \frac{\tau_s(v_{s_{sim}})}{\sigma_0} \frac{A_c\left(T_{s_{sim}}, v_{s_{sim}}, \sigma_0\right)}{A_0} \tag{8}$$

where τ_s is the local shear stress acting in the real contact area and depends on the sliding velocity and the compound properties; σ_0 is the simulation nominal contact pressure, one of the GrETA model input; A_c is the effective contact area and A_0 is the nominal one.

The shear stress is the local force needed to break the adhesive bounds at the road tire interface. The effective contact area is a variable difficult to determine and many empirical formulations are advanced in the scientific literature [3, 4, 23] based on specific approaches. The main reason may lie in the lack of a reliable experimental method for its estimation.

For the adhesive friction contribution, the following empirical relationship has been considered based on the simplified Persson's theory [4], where the Gauss error function, denoted as erf, is considered as follows:

$$\frac{A_c(T, v_s)}{A_0} = \mathrm{erf}\left(\sigma_0 \frac{1-v^2}{\sqrt{m2}\, E^*(T, v_s)}\right) \tag{9}$$

In equation (9), v is the Poisson's coefficient, which is approximated as a constant value (~0.48), and E^* is the dynamic complex modulus of the tread compound sliding on the asphalt at the velocity v_s and the temperature T.

Hysteresis model

The hysteretic contribution, due to the cyclic deformation of a viscoelastic material, is evaluated with a power balance defined by Etienne-David [24], defined in equation (10), whose terms are calculated starting from a multiscale simplified approach based on the Hertzian contact mechanics theory, leading to improved results if compared to the ones described in [8]:

$$\mu_{hys} \sigma_0 A_0 v_{s_{sim}} = \int_V w(x, y, z, t) dV \tag{10}$$

in which μ_{hys} is the unknown grip term depending on the stress field and strain rate. The quantity $w(x,y,z,t)$ represents the power loss at each point of the deformed elementary volume at the time t. Hypothesizing a constant sliding velocity, each stress-strain cycle can be considered as performed in a period equal to:

$$T = \frac{\lambda}{v_s} \tag{11}$$

This power loss can be expressed for the entire compound volume considered and equaled to the dissipation linked to friction forces:

$$W_{diss}(t) = \int_V w(x, y, z) dV = \int_V \sigma(x, y, z, t) \frac{d\varepsilon}{dt}(x, y, z, t) dV \tag{12}$$

Assuming the external solicitation linked to sinusoidal road asperities as periodic, the strain can be expressed as follows. Equation 13 and 14 describe a shape of the strain (ϵ) and stress (σ). E_0 and Σ_0 are generic amplitude.

$$\varepsilon(t) = \mathrm{E}_0 \sin(\omega t) \tag{13}$$

The associated stress field in case of viscoelastic material is:

$$\sigma(t) = \Sigma_0 \sin(\omega t + \delta) \tag{14}$$

Therefore, the equation (12) can be rewritten as:

$$W_{diss} = \int_V \left[\frac{1}{T}\int_0^T \Sigma_0 \sin(\omega t + \delta)\, \omega \mathrm{E}_0 \cos(\omega t)\, dt\right] dV \tag{15}$$

The dissipated power is evaluated for each wavelength considered in hysteretic grip calculation in case of multi-contact grip model. Hence, the total dissipated power in the control volume is determined as follows:

$$W_{tot} = \sum_{i=1}^{M} w_{p,i} \cdot W_{diss,i} \tag{16}$$

The total hysteretic contribution is consequentially defined as:

$$\mu_{hys}\sigma_0 A_0 v_s = W_{tot} = \sum_{i=1}^{M} w_{p,i} \cdot W_{diss,i} \tag{17}$$

Once explained the adhesion and friction contribution determined by means of the GrETA model, the total grip can be expressed as follows:

$$\mu = \mu_{adh} + \mu_{hys} \tag{18}$$

It must be pointed out that the equation (18), which states that the adhesion and hysteretic contribution can be treated separately, is an approximation adopted into the specific model.

EXPERIMENTAL ANALYSIS: BRITISH PENDULUM EVO

Test Bench Description

The tribological bench used for the calibration and the validation of the model is available at the Tire Lab of the Department of Industrial Engineering (DII) at University of Naples Federico II and it is based on an evolved version of the classic British Pendulum tester for skid resistance measurements. Contrariwise the old configuration of the BP-evo [16, 25], which was developed during previous collaborations of the DII with tire manufacturing companies, this updated version has a new configuration of the load cell capable of reading the three force channels (the tangential and the normal ones) according to the sensor reference system. The BP-evo and its conceptual scheme are shown in Fig. 2.

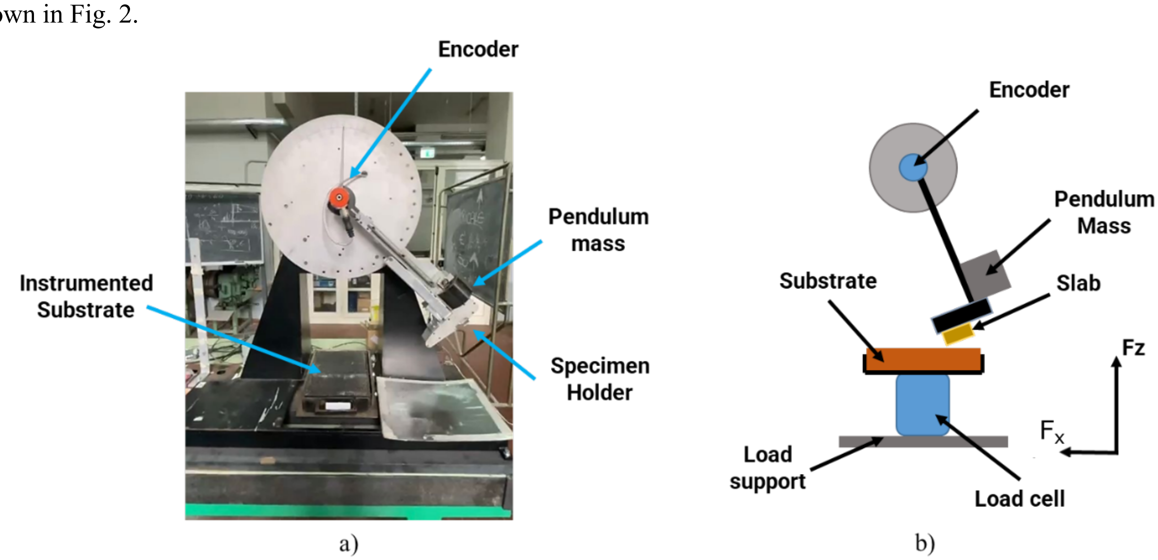


FIGURE 2. a) British Pendulum evolved – b) Test bench scheme

As represented in these figures, the tri-axial load cell is fixed on a rigid support and the rough substrate is fixed above the sensor. The cell is positioned in order to acquire positive normal load F_z and negative tangential force Fx along the sliding direction; moreover, the positioning is centered with respect to the plane of the pendulum motion allowing to get neglectable values of F_Y. The previous layout of the BP-evo included the load cell mounted on the pendulum arm, making the acquired signals processing more complicated due to inertial forces calculation and detraction from the global values. An encoder is installed in the revolute joint to measure the angular speed of the pendulum arm, on which is fixed a mass so that the sliding body exhibits enough potential energy to win the frictional resistance forces.

The 20x20 mm specimens are usually obtained from tire tread or compound slabs and then fixed on the holder, which can be regulated so that the material correctly slides. The distance between the revolute joint and the substrate is adjustable thanks to a regulation mechanism behind the pendulum. This regulation is fundamental in order to set the proper sliding distance.

The forces and encoder signals are acquired by an A/D board and processed in Matlab environment to convert from Volt to N/m and rad/s respectively. An adjustable release mechanism of the pendulum arm is available in order to change the starting position and perform a set of measurements with high repeatability. It is important to change the drop position so that different sliding velocity ranges can be reached during the motion and therefore, the friction coefficient values with respect to sliding velocity can be analyzed in the post-processing phase.

GRETA MODEL PARAMETRIZATION

Material

The viscoelastic characteristics of the tread compound are one of the main inputs for the GrETA model. They can be estimated performing a DMA on specific specimens coming from compound "slabs", which are usually produced with a manufacturing process not totally similar to tire one, or by means of VESevo non-destructive testing [15], which is an innovative device for non-destructive viscoelasticity tire analysis.

The viscoelastic storage modulus and loss factor can be provided in terms of temperature sweep master curves at a certain reference frequency (1 Hz), otherwise, in the form of frequency sweep data. In both cases, these properties are transformed thanks to the WLF relationship [13, 26]

$$\log\left(\frac{f_2}{f_1}\right) = \frac{T^* - T_1}{\Delta T} \tag{19}$$

In which a common ΔT value, identifiable by means of DMA tests at different frequencies, is about 8°C.

The evaluation of the viscoelasticity at different temperatures and frequencies is fundamental because it is necessary to consider the compound behavior variation in response to the tire-road contact: different wavelengths excite the material at different frequencies linked to the asperity wavelength by this expression:

$$f_i = \frac{v_s}{\lambda_i} \tag{20}$$

where, f_i is the excitation frequency linked to the spatial frequency of the asperities λ_i, through the sliding velocity of the tread block, v_s. In Fig. 3 the viscoelastic properties shift from the DMA reference curve at 1 Hz (black full line) to higher frequencies due to sliding on the roughness surface is shown. The values on x-axis are hidden due to the industrial confidentiality agreement with the compound manufacturer. As noticeable, the application of the WLF relationship not only shifts horizontally the viscoelastic master curves but slightly changes their shape (Fig. 3) affecting the grip estimation in temperature range of interest.

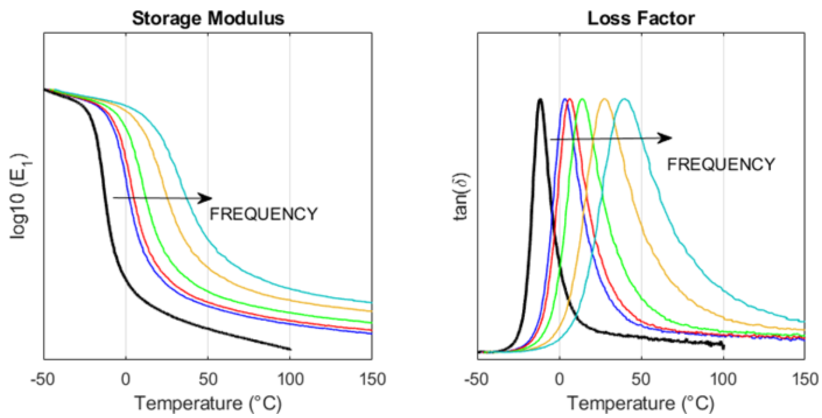


FIGURE 3. Application of the WLF frequency shift from the black reference curve (DMA 1 Hz)

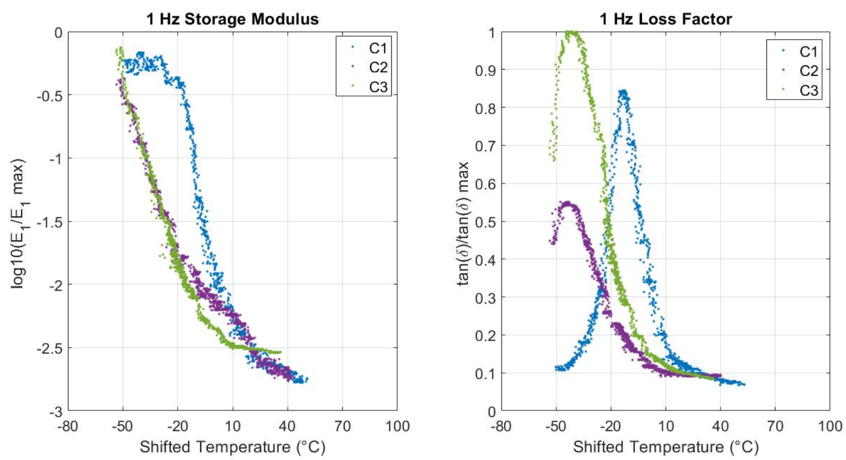


FIGURE 4. VESevo 1Hz normalized master curves for compounds of interest

Road Surface

As described in the section "Mathematical model", the first class of input variables is related to the roughness profile parameters. In this work, the analysis of the grip coefficient, carried out through the GrETA model and the experimental bench, has been carried out with asphalt specimens of 200x250 mm (Fig. 5).

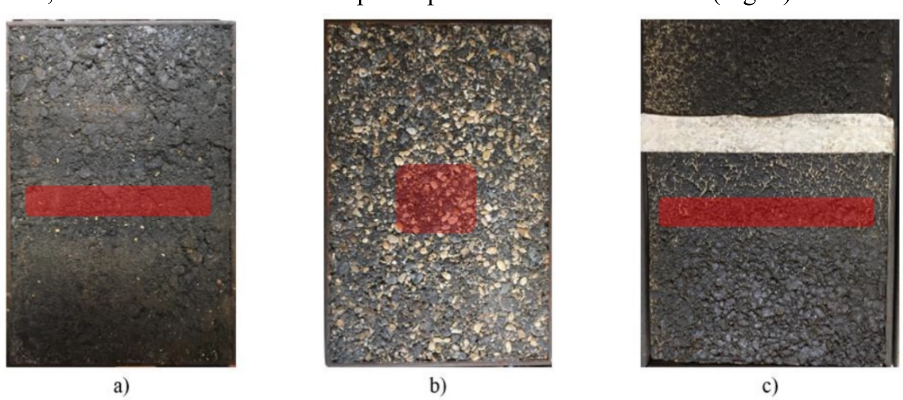


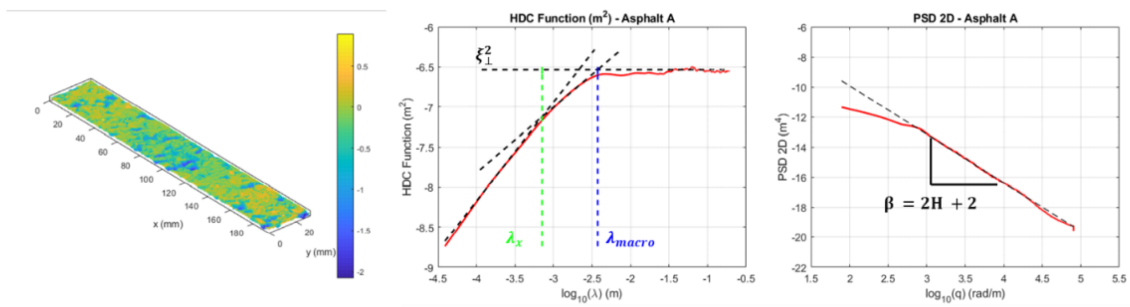
FIGURE 5. a) Asphalt A – b) Asphalt B – c) Asphalt C. The red areas match with the scanned zones

For each surface, the roughness profile scans have been analyzed by means of HDC and PSD 2D functions. The input parameters for the GrETA model are outlined in Table 1.

TABLE 1. Roughness Parameters

Asphalt	Ra (mm)	λ_{macro} (mm)	λ_X (mm)	$\xi_\perp$ (mm)	H (-)	K (-)	S_k (-)
Asphalt A	0.275	3.60	0.9	0.5	0.625	5.10	-1.43
Asphalt B	0.799	4.20	1.4	1.3	0.955	2.17	-0.168
Asphalt C	0.513	3.58	1.0	1.1	0.705	6.28	-1.72

The 3D scan of a portion of the asphalt specimen A, together with the corresponding HDC and PSD functions are shown in Fig. 6. As noticeable, the main roughness parameters of Table 1 can be extrapolated from the above diagrams to feed up the tire contact mechanics equations implemented into the GrETA model for hysteretic and adhesive contributions to friction coefficient.



a) b) c)

FIGURE 6. Asphalt A: a) 3D scan – b) HDC Function – c) PSD 2D Function

VALIDATION AND RESULTS

In this section the friction coefficient results on different compound specifications obtained from BP evo test bench have been compared to GrETA model values to verify the trustworthiness of the simulations. The proposed contact mechanics model has been fed up with the road parameters in Table 1, the viscoelastic properties obtained with VESevo measurements and displayed in Fig. 4, the WLF parameters calculated from the DMA and VESevo data.

In Fig. 7-9, the comparisons between GrETA and BP-evo measurements for the compounds analysed on three different textures are shown. The model can reproduce quite faithfully the trends of the experimental measurements both in different asphalt and temperature conditions, especially in the velocity range of (1, 2) m/s. The model estimation for the compound C1 and C3, for the temperatures of 45 and 70° are very close because, how it can be seen in Fig. 4, the compound working range is hotter than the tan(δ) function peak, and it's working on the plateau region. In some cases, a mismatch between data and model occurs at low velocity (< 1 m/s) and this may be due to the lower reliability of BP-evo because small contact lengths (≤ 5 cm) lead to stick slip phenomena that heavily affect the BP dynamics and alter the test results.

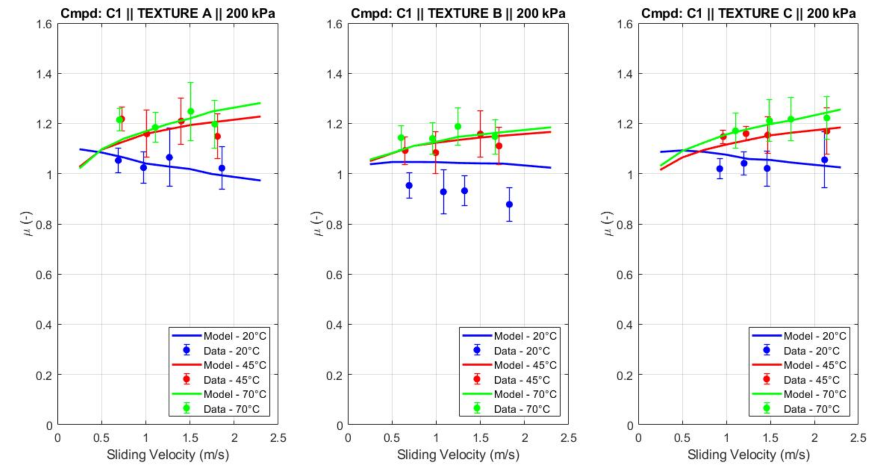


FIGURE 7. Experimental friction results compared to GrETA simulation - Compound C1

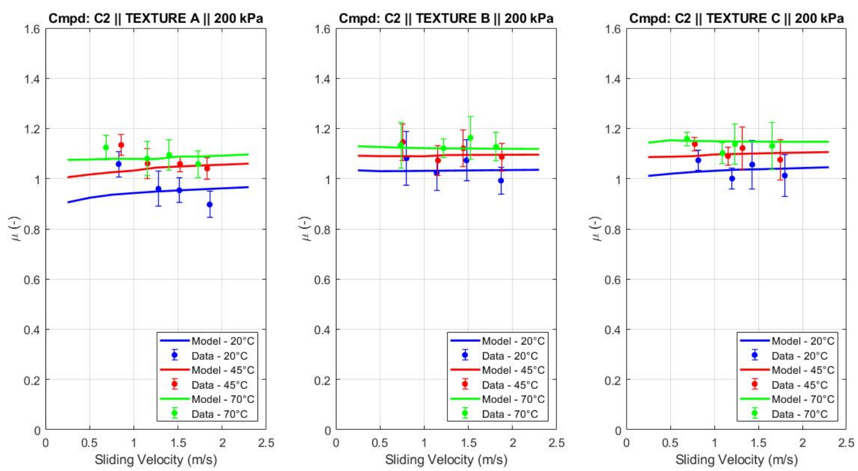


FIGURE 8. Experimental friction results compared to GrETA simulation - Compound C2

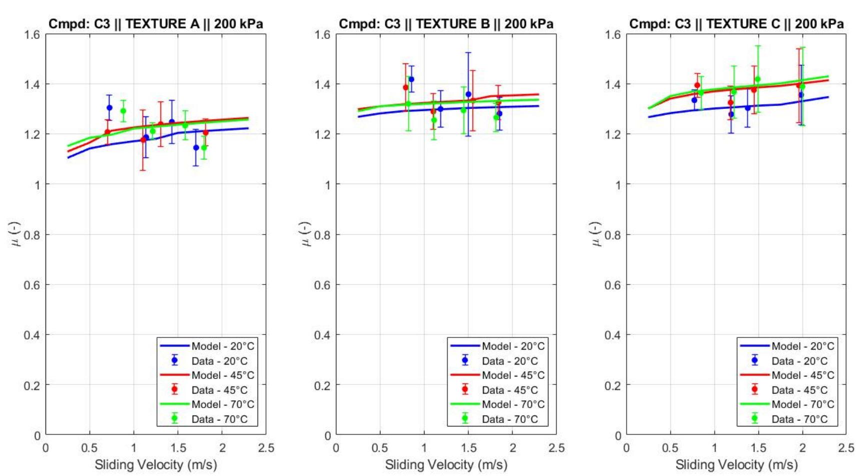


FIGURE 9. Experimental friction results compared to GrETA simulation - Compound C3

The GrETA model has been also capable of recognizing the frictional behaviour changes of compound C1: the experimental data, such as the model results, exhibit a slightly increasing trend towards the sliding velocity, which may be since a relevant hysteretic contribution occurs at high frequencies, at both 45°C and 70°C; contrariwise, a slightly decrement is noticeable at 20°C of model and BP-evo data.

CONCLUSION

In this paper the authors' aim has been to investigate the friction between tire tread specimens and road surface in dry condition and at three different working temperature on different compounds characterized by different viscoelastic properties. Such properties have been evaluated by an innovative non-destructive device developed thanks to the support of the Applied Mechanics research team of the Industrial Engineering Department of University Federico II. The viscoelastic characterization of tire compounds aims to the comprehension of the contact phenomena between tyre tread and road. For this purpose, the road profile analysis in terms of macro and micro asperity scales has been analysed. To analyse the phenomena concerning the multi-contact problem between viscoelastic materials and rough surfaces the GrETA model has been described to predict the friction variations induced by different tread compounds characterized by means of VESevo. Finally, the results of the simulation obtained by GrETA model, have been compared to the same carried out by the experimental test bench called British Pendulum EVO.

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